COMPUTER-AIDED DETERMINATION OF EQUIVALENT CIRCUITS FOR WAVEGUIDE DISCONTINUITIES

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Abstract

The use of a digital computer to convert a set of measured standing wave null positions versus sliding short positions to an equivalent circuit for a discontinuity in a uniform waveguide is discussed. This method when employed to interpret data indicates the presence of systematic or random errors and averages the accumulated information directly.

General Considerations

A discontinuity in a waveguide (Figure 1a) can be represented by the equivalent circuit[1] described in Figure 1b. The unknown quantities to be determined are the lengths, L_1 and L_2, and the admittance, y, which are dependent on the geometry of the guide, the discontinuity, the choice of reference plane, and the polarization of wave vector. In the following discussion, a discontinuity that is symmetrical about the reference plane will be assumed. Nevertheless, the technique to be described will be applicable to an asymmetrical discontinuity. The lengths, L's, that will be found, include the foreshortening or lengthening of the guide produced by the discontinuity. It is readily seen that the placing of the reference plane at a point other than the plane of symmetry will result in the same admittance, y, but will cause a corresponding shift in the lengths, L's.

The measurements are made with the arrangement illustrated in Figure 2a. For each short position, J, the corresponding standing wave null position K, is measured. A typical plot of data might appear as shown in Figure 2b. The true curve probably would not pass through all the measured points because of measurement inaccuracies. The objective here is to determine those values of y and L's that "best fit" the experimental data. The peak-to-peak amplitude, A, of the curve indicates the magnitude of y, whereas the lengths, L's, shifts the mean axis vertically and curve laterally.[2] The curve will make one cycle for each half wavelength movement of the short.

The "best fitting" process is described next and is based on the "method of least squares" technique discussed in the literature.[3]

Method of Least Square

For a given short position, the voltage standing wave pattern (VSWP) might appear as shown in Figure 3a. It is seen that the VSWP is discontinuous across the plane of symmetry due to the presence of the discontinuity. If, however, the VSWP on the source side is imagined as continuous across the discontinuity, the apparent null position just to the right of the discontinuity is displaced from the first actual VSWP null. The shift, $D_m(i)$, which is obtained through measurement is defined as:

$$D_m(i) = J(i) - K(i) \tag{1}$$

where i denotes the i^{th} value out of N measurements. On the other hand, the shift can also be calculated if y, L_1 and L_2 were known, as well as the measured J(i). To distinguish from $D_m(i)$, let us denote it by $D_c(i)$. Referring to Figure 3b, and assuming the circuit is dissipationless and the short has a unity reflection coefficient, it is easy to show that $D_c(i)$ is an explicit function of y, L_1, L_2 and J(i).

$$D_c(i) = f\left[y,\ L_1,\ L_2,\ J(i)\right] \tag{2}$$

$$= \frac{1}{\beta}\tan^{-1}\frac{1+[\tan\beta\,(J(i)-L_1)][y-\mathrm{Cot}\,\beta\,(J(i)+L_2)]}{[y-\mathrm{Cot}\,\beta\,(J(i)+L_2)]-[\tan\beta\,(J(i)-L_1)]} \tag{3}$$

where $\beta = 2\pi/\lambda_g$, λ_g is the guide wavelength

$y = Y/Y_0$ is the normalized susceptance.

The residuals, E(i), are defined as the difference between $D_m(i)$ and $D_c(i)$; i.e.

$$E(i) = D_m(i) - D_c(i). \tag{4}$$

Using the Taylor's Series expansion, we may approximate $D_c(i)$ as

$$\begin{aligned} D_c(i) = {} & f(\tilde{y},\ \tilde{L}_1,\ \tilde{L}_2,\ J(i)) + \left[f_y\Delta_y + f_{L_1}\Delta L_1 + f_{L_2}\Delta L_2\right] \\ & + 1/2\left[f_{yy}(\Delta y)^2 + f_{L_1L_1}(\Delta L_1)^2 + f_{L_2L_2}(\Delta L_2)^2\right] \\ & + \left[f_{yL_1}\Delta y\Delta L_1 + f_{L_1L_2}\Delta L_1\Delta L_2 + f_{L_2y}\Delta L_2\Delta y\right] \\ & + \text{higher order terms in } \Delta y \text{ and } \Delta L\text{'s}. \end{aligned} \tag{5}$$

where f_y, f_{yy}, ...etc. are the first and second partial derivatives of f(y, L_1, L_2, J(i)) and are functions of J(i); $\tilde{y}$, $\tilde{L}_1$ and $\tilde{L}_2$ are approximate values obtained initially by an estimate; and Δy, ΔL_1 and ΔL_2 are corrections such that the actual values of y, L_1 and L_2 are:

$$y = \tilde{y} + \Delta y;\ L_1 = \tilde{L}_1 + \Delta L_1;\ L_2 = \tilde{L}_2 + \Delta L_2 \tag{6}$$

Their values are to be found so that the sum of the squares of the residuals is a minimum, which results in the following set of normal equations:

$$\sum_1^N E(i)\frac{\partial}{\partial \Delta y}E(i) = 0,\quad \sum_1^N E(i)\frac{\partial}{\partial \Delta L_1}E(i) = 0,\quad \sum_1^N E(i)\frac{\partial}{\partial \Delta L_2}E(i) = 0 \tag{7}$$

To develop the equations of (7), the series of equation (5) is truncated, which will lead only to an approximate determination of the circuit parameters y, L_1 and L_2. If these are not sufficiently accurate, they might be improved by being used as the next estimate and repeating the process of equations (4) through (7) until a satisfactory result is attained. It was found that the neglect of the second and higher order terms of (5), frequently led to a slow convergence and sometimes no convergence at all. Including the second order terms greatly improved the rate and the range of convergence. The complexity of the equations (7) increased rapidly with the order of approximation of equation (5). Therefore, it was decided to truncate (5) after the second order.

Substituting the second order approximation of E(i) and its derivatives into equation (7) and rearranging, yields matrix equation 8:

$$\begin{bmatrix} \Delta y \\ \Delta L_1 \\ \Delta L_2 \end{bmatrix} = \begin{bmatrix} A & F & G \\ F & B & H \\ G & H & C \end{bmatrix}^{-1} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \qquad (8)$$

where

$$A = \sum_1^N \left[f_y^2 - Ef_{yy}\right] ; \quad B = \sum_1^N \left[f_{L_1}^2 - Ef_{L_1L_1}\right] ; \quad C = \sum_1^N \left[f_{L_2}^2 - Ef_{L_2L_2}\right]$$

$$F = \sum_1^N \left[f_y f_{L_1} - Ef_{yL_1}\right] ; \quad G = \sum_1^N \left[f_y f_{L_2} - Ef_{yL_2}\right] ; \quad H = \sum_1^N \left[f_{L_1} f_{L_2} - Ef_{L_1L_2}\right]$$

$$E_1 = \sum_1^N f_y \quad ; \quad E_2 = \sum_1^N f_{L_1} \quad ; \quad E_3 = \sum_1^N f_{L_2}$$

which may be solved for Δy, ΔL_1 and ΔL_2 to obtain the corrected value of y, L_1 and L_2.

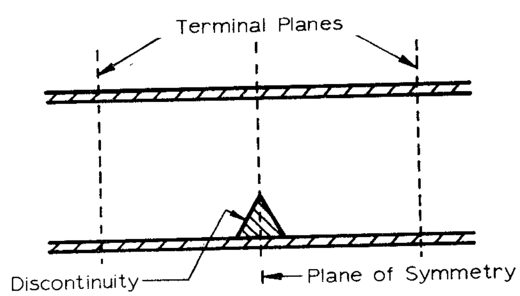
Program and Results

A computer program was written to implement the preceding analysis, which is sufficiently fast and economical to run on a time-shared terminal. For example, each iteration of a 12-point data set required approximately 2 CPU seconds with IBM's Call 360 Basic Time Share System. The number of iterations required varied widely, i.e., 3 to 20, depending on the accuracy of the initial guess and on the consistency of the measured data. The determination of the small discontinuities generally converged much more slowly. If an inaccurate initial guess was used or the data was too inconsistent, the process would not converge.[4] This fact might be an advantage since it prevents one from accepting poor data.

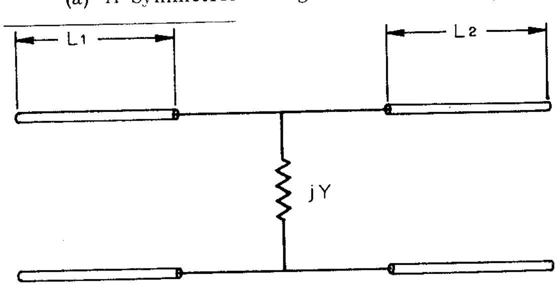
As an illustration, two typical computer printouts are reproduced and presented in Figure 4, corresponding to different input data. The number of iterations is different in each case, indicating the effect of initial guess accuracy and measured data consistency on the rate of convergence.

References

1. Marcuvitz, N.; "Waveguide Handbook," M.I.T. Rad. Lab. Series, Volume 10, McGraw-Hill, 1951, p. 108.
2. Ginzton, E. L.; "Microwave Measurements," McGraw-Hill, 1957, p. 278.
3. Sokolninoff, I. S. & E. S.: "Higher Mathematics for Engineers and Physicists," McGraw-Hill, 1941, p. 536.
4. Box, G. E. P. & Gwilym, M. J.; "Time Series Analysis - Forecasting and Control," Holden-Day, 1970, p. 233.

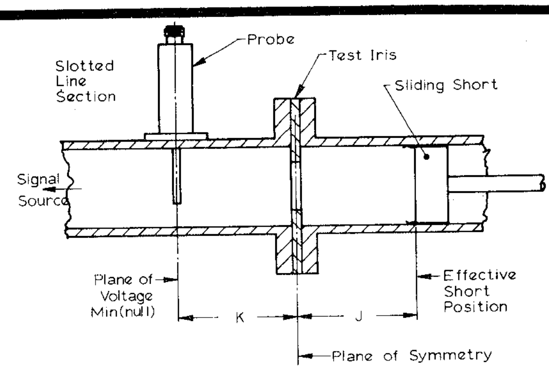


(a) A Symmetric Waveguide Discontinuity

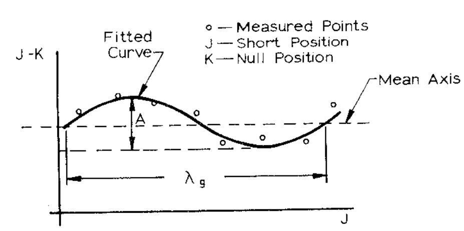


(b) Equivalent Circuit

Figure 1

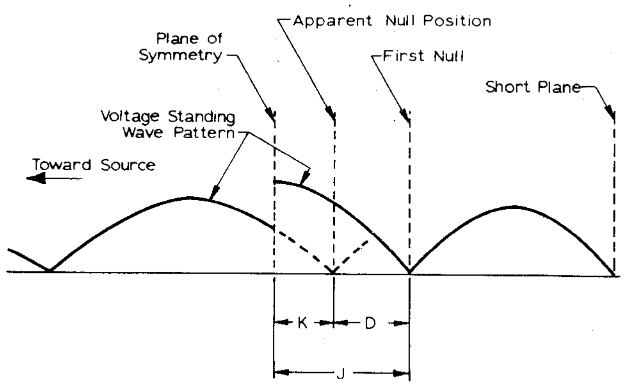


(a) Measurement Arrangement

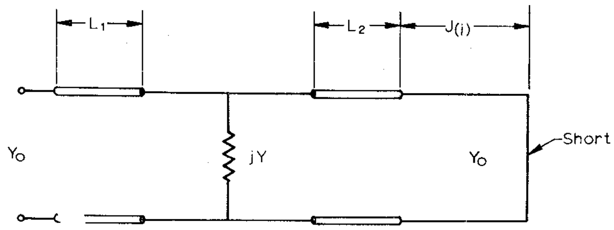


(b) Plot of Measured Data

Figure 2



(a) Voltage Standing Wave Pattern



(b) Equivalent Circuit With Sliding Short

Figure 3

```
INITIAL VALUES:B,L1,L2:  -0.053      0.200      -0.200
INDEX    RESIDUAL
  1       -0.012
  2       -0.020
  3       -0.015
  4       -0.014
  5       -0.007
  6       -0.009
  7       -0.004
  8       -0.005
  9       -0.010
 10       -0.011
 11       -0.009
ERROR: MEAN: -1.05535E-02      RMS:  4.36954E-03
CORRECT B,L1,L2 BY:  -0.0094    -0.0768      0.0892

NEW VALUES:B,L1,L2:  -0.062      0.123      -0.111
     SECOND ITERATION
NEW VALUES:B,L1,L2:  -0.054      0.123      -0.113
     THIRD ITERATION
NEW VALUES:B,L1,L2:  -0.054      0.125      -0.115
     FOURTH ITERATION
INDEX    RESIDUAL
  1       -0.001
  2       -0.006
  3       -0.000
  4       -0.002
  5        0.002
  6       -0.003
  7        0.002
  8        0.002
  9        0.001
 10        0.003
 11        0.005
ERROR: MEAN:  2.41467E-07      RMS:  3.04466E-03
CORRECT B,L1,L2 BY:   0.0000     0.0000     -0.0000

NEW VALUES:B,L1,L2:  -0.054      0.125      -0.115
```

(a) Computer Printout, Case I

```
INITIAL VALUES:B,L1,L2:  -0.013      -0.400      0.300
INDEX    RESIDUAL
  1       -0.072
  2       -0.075
  3       -0.096
  4       -0.099
  5       -0.095
  6       -0.085
  7       -0.057
  8       -0.068
  9       -0.087
 10       -0.092
ERROR: MEAN: -8.27019E-02      RMS:  1.32369E-02
CORRECT B,L1,L2 BY:  -0.0149    -0.0922      0.1787

NEW VALUES:B,L1,L2:  -0.028     -0.492      0.479
     SECOND ITERATION
NEW VALUES:B,L1,L2:  -0.055     -0.070      0.064
     THIRD ITERATION
NEW VALUES:B,L1,L2:  -0.052      0.057     -0.060
     FOURTH ITERATION
NEW VALUES:B,L1,L2:  -0.051      0.107     -0.110
     FIFTH ITERATION
NEW VALUES:B,L1,L2:  -0.050      0.113     -0.116
     SIXTH ITERATION
INDEX    RESIDUAL
  1        0.000
  2       -0.005
  3       -0.014
  4        0.001
  5       -0.004
  6       -0.007
  7        0.013
  8        0.006
  9        0.002
 10        0.007
ERROR: MEAN:  8.68738E-07      RMS:  7.34656E-03
CORRECT B,L1,L2 BY:   0.0000     0.0002     -0.0002

NEW VALUES:B,L1,L2:  -0.050      0.113     -0.116
```

(b) Computer Printout, Case II

Figure 4

Notes